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Lepton flavor violating signals of a little Higgs model at high energy linear e^+e^- colliders

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Abstract. The littlest Higgs (LH) model predicts the existence of the doubly charged scalars $\Phi^{\pm\pm}$, which generally have large flavor changing couplings to leptons. We calculate the contributions of $\Phi^{\pm\pm}$ to the lepton flavor violating (LFV) processes $l_i \to l_j\gamma$ and $l_i \to l_j l_k l_k$, and we compare our numerical results with the current experimental upper limits on these processes. We find that some of these processes can give severe constraints on the coupling constant Y_{ij} and the mass parameter M_Φ. Taking into account the constraints on these free parameters, we further discuss the possible lepton flavor violating signals of $\Phi^{\pm\pm}$ at high energy linear e^+e^- collider (ILC) experiments. Our numerical results show that the possible signals of $\Phi^{\pm\pm}$ might be detected via the subprocesses $e^\pm e^\pm \to l^\pm l^\pm$ in future ILC experiments.

1 Introduction

It is well known that the individual lepton numbers L_e, L_μ, and L_τ are automatically conserved and tree level lepton flavor violating (LFV) processes are absent in the standard model (SM). However, neutrino oscillation experiments have made one believe that neutrinos are massive and oscillate in flavor, which presently provides the only experimental hints of new physics and implies that the separated lepton numbers are not conserved [1–3]. Thus, the SM requires some modification to account for the pattern of neutrino mixing, in which the LFV processes are allowed. The observation of LFV signals in present or future high energy experiments would be a clear signature of new physics beyond the SM.

Some of the popular specific models beyond the SM generally predict the presence of new particles, such as new gauge bosons and new scalars, which may naturally lead to tree level LFV coupling. In general, these new particles could enhance the branching ratios for some LFV processes and perhaps bring them past the observability threshold of the present and next generations of collider experiments. Furthermore, non-observability of these LFV processes may lead to strong constraints on the free parameters of new physics. Thus, studying the possible LFV signals of new physics in various high energy collider experiments is very interesting and is much needed.

Little Higgs models [4, 5] (the former reference gives a recent review) employ an extended set of global and gauge symmetries in order to avoid one-loop quadratic divergences and thus provide a new method to solve the hierarchy between the TeV scale of possible new physics and the electroweak scale $\nu = 246\,\text{GeV} = (\sqrt{2}G_\text{F})^{-\frac{1}{2}}$. In this kind of models, the Higgs boson is a pseudo-Goldstone boson of a global symmetry that is spontaneously broken at some high scales. Electroweak symmetry breaking (EWSB) is induced by radiative corrections leading to a Coleman–Weinberg type of potential. Quadratic divergence cancelation of radiative corrections to the mass of the Higgs boson is due to contributions from new particles with the same spin as the SM particles. This type of models can be regarded as one of the important candidates of new physics beyond the SM.

The littlest Higgs model (LH) [6] is one of the simplest and one of the phenomenologically viable models that realizes the little Higgs idea. Recently, using the fact that the LH model contains a complex triplet Higgs boson Φ [7–10] we discussed the possibility of introducing lepton number violating interactions and of generating a neutrino mass in the little Higgs scenario. Reference [9] has shown that the most satisfactory way of incorporating neutrino masses is to include a lepton number violating interaction between the triplet scalars and lepton doublets. The tree level neutrino masses are mainly generated by the vacuum expectation value (VEV) ν' of the complex triplet Φ, which does not affect the cancelation of quadratic divergences in the Higgs mass. The neutrino masses can be given by the term $Y_{ij}\nu'$, in which Y_{ij} (i, j are generation indices) is the Yukawa coupling constant. As long as the triplet VEV ν' is restricted to be extremely small, the value of Y_{ij} is of one of a natural order of magnitude, i.e. $Y_{ij} \approx 1$, which might produce large contributions to some of LFV processes [10, 11].

The aim of this paper is to study the contributions of the LFV couplings predicted by the LH model to the LFV processes $l_i \to l_j\gamma$ and $l_i \to l_j l_k l_k$ and compare our numerical results with the present experimental bounds on these

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LFV processes, and the aim is to see whether constraints on the free parameter Y_{ij} can be obtained. We further calculate the contributions of the LH model to the LFV processes $e^{\pm}e^{\pm} \to l_i^{\pm} l_j^{\pm}$ and $e^+e^- \to l_i^{\pm} l_j^{\pm} (l_i \text{ or } l_j \neq e)$, and we discuss the possibility of detecting the LFV signals of the LH model via these processes in future high energy linear e^+e^- collider (ILC) experiments.

This paper is organized as follows. Section 2 contains a short summary of the relevant LFV couplings of the scalars (the doubly charged scalars $\Phi^{\pm\pm}$, the charged scalars $\Phi^{\pm}$, and the neutral scalar Φ^0) to lepton doublets. The contributions of these LFV couplings to the LFV processes $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_k$ are calculated in Sect. 3. Using the current experimental upper limits on these LFV processes, we try to give constraints on the coupling constant Y_{ij} in this section. Section 4 is devoted to the computation of the production cross sections of the LFV processes $e^{\pm}e^{\pm} \to l_i^{\pm} l_j^{\pm}$ and $e^+e^- \to l_i^{\pm} l_j^{\pm}$ induced by the doubly charged scalars $\Phi^{\pm\pm}$. Some phenomenological analyses are also included in this section. Our conclusions are given in Sect. 5.

2 The LFV couplings of the triplet scalars

The LH model [6] consists of a non-linear σ model with a global SU(5) symmetry and a locally gauged symmetry $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$. The global SU(5) symmetry is broken down to its subgroup SO(5) at a scale $f \sim$ TeV, which results in 14 Goldstone bosons (GBs). Four of these GBs are eaten by the gauge bosons ($W_H^{\pm}, Z_H, B_H$), resulting from the breaking of $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$, and giving them masses. The Higgs boson remains as a light pseudo-Goldstone boson, and the other GBs give masses to the SM gauge bosons and form a scalar triplet Φ. The complex triplet Φ offers a chance to introduce lepton number violating interactions in the theory.

In the context of the LH model, the lepton number violating interaction that is invariant under the full gauge group can be written as [9, 11]

$$\mathcal{L} = -\frac{1}{2} Y_{ij} \left(L_i^{\mathrm{T}}\right)_{\alpha} \Sigma_{\alpha\beta}^{*} C^{-1} \left(L_j^{\mathrm{T}}\right)_{\beta} + \text{h.c.}\,, \tag{1}$$

where i and j are generation indices, α and β $(= 1, 2)$ are SU(5) indices, and $L^{\mathrm{T}} = (l_L, \nu_L)$ is a left handed lepton doublet. Y_{ij} is the Yukawa coupling constant, and C is the charge-conjugation operator. Because of the non-linear nature of $\Sigma_{\alpha\beta}^{*}$, this interaction can give rise to a mass matrix for the neutrinos as follows:

$$M_{ij} = Y_{ij} \left(\nu' + \frac{\nu^2}{4f}\right) . \tag{2}$$

One can see from (2) that, if we would like to stabilize the Higgs mass and at the same time ensure neutrino masses consistent with the experimental data [12–14], the coupling constant Y_{ij} must be of the order of 10^{-11}, which is unnaturally small. However, it has been shown [7–9] that the lepton number violating interaction only involving the complex scalar triplet Φ can give a neutrino mass matrix $M_{ij} = Y_{ij}\nu'$. Considering the current bounds on the neutrino mass [12–14], we should have

$$Y_{ij}\nu' \sim 10^{-10}\,\mathrm{GeV}\,. \tag{3}$$

Thus, the coupling constant Y_{ij} may naturally be of order one or at least need not be unnaturally small, provided the VEV ν' of the triplet scalar Φ is restricted to be extremely small.

In this scenario, the triplet scalar Φ has LFV couplings to the left handed lepton pairs, which can be written as [9]

$$\mathfrak{L}_{\mathrm{LFV}} = Y_{ij} \Bigg[l_{Li}^{\mathrm{T}} C^{-1} l_{Lj} \Phi^{++} + \frac{1}{\sqrt{2}} \left(\nu_{Li}^{\mathrm{T}} C^{-1} l_{Lj} + l_{Li}^{\mathrm{T}} C^{-1} \nu_{Lj}\right) \Phi^{+} + \nu_{Li}^{\mathrm{T}} C^{-1} \nu_{Lj} \Phi^0 \Bigg] + \text{h.c.} \tag{4}$$

Considering these LFV couplings, [9] has investigated the decays of the scalars $\Phi^{\pm\pm}$ and $\Phi^{\pm}$ and has found that the most striking signature comes from the doubly charged scalars $\Phi^{\pm\pm}$. The constraints on the coupling constant Y_{ij} and the triplet scalar mass parameter M_{Φ} coming from the muon anomalous magnetic moment a_{μ} and the LFV process $\mu^- \to e^+e^-e^-$ are studied in [11]. In the next section, we will calculate the contributions of the charged scalars $\Phi^{\pm\pm}$ and $\Phi^{\pm}$ to the LFV processes $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_k$.

3 The charged scalars and the LFV processes $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_k$

The observation of neutrino oscillations [1–3] implies that the individual lepton numbers $L_{e,\mu,\tau}$ are violated, suggesting the appearance of LFV processes, such as $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_k$. The branching ratios of these LFV processes are extremely small in the SM with right handed neutrinos. For example, [15, 16] have shown that $\mathrm{Br}(\mu \to e\gamma) < 10^{-47}$. Such a small branching ratio is unobservable.

The present experimental upper limits on the branching ratios $\mathrm{Br}(\mu \to e\gamma)$ [17], $\mathrm{Br}(\mu \to 3e)$ [18], $\mathrm{Br}(\tau \to e\gamma)$ [19], $\mathrm{Br}(\tau \to \mu\gamma)$ [20], and $\mathrm{Br}(\tau \to l_i l_k l_k)$ [21] are given in Table 1. Future experiments with increased sensitivity can reduce these current limits by a few orders of magnitude (see, e.g., [22]). In this section, we will use these data to give the constraints on the free parameters Y_{ij} and M_{Φ}.

The LFV couplings of the charged scalars Φ^{--} and Φ^{-} given in (4) may lead to the LFV radiative decays $l_i^- \to l_j^- \gamma$ at the one-loop level mediated by the exchange of the charged scalars Φ^{--} and Φ^{-}, as shown in Fig. 1. For the doubly charged scalar Φ^{--}, the photon can be attached either to the internal lepton line or to the scalar line. For the charged scalar Φ^{-}, the photon can only be attached to the scalar line [23, 24].

Table 1. The current experimental upper limits on the branching ratios of some LFV processes and the corresponding upper constraints on the free parameters

Decay process	Current limit	Bound (GeV^{-4})
$\mu \to e\gamma$	1.2×10^{-11} [17]	–
$\tau \to e\gamma$	1.1×10^{-7} [19]	–
$\tau \to \mu\gamma$	6.8×10^{-8} [20]	–
$\mu \to 3e$	1.0×10^{-12} [18]	$\mid Y_{\mu e} Y_{ee}^* \mid^2 / M_\Phi^4 \leq 2.2 \times 10^{-19}$
$\tau \to 3e$	2.0×10^{-7} [21]	$\mid Y_{\tau e} Y_{ee}^* \mid^2 / M_\Phi^4 \leq 2.4 \times 10^{-13}$
$\tau \to 2\mu e$	3.3×10^{-7} [21]	$\mid Y_{\tau e} Y_{\mu\mu}^* \mid^2 / M_\Phi^4 \leq 8.1 \times 10^{-13}$
$\tau \to 2e\mu$	2.7×10^{-7} [21]	$\mid Y_{\tau\mu} Y_{ee}^* \mid^2 / M_\Phi^4 \leq 6.6 \times 10^{-13}$
$\tau \to 3\mu$	1.9×10^{-7} [21]	$\mid Y_{\tau\mu} Y_{\mu\mu}^* \mid^2 / M_\Phi^4 \leq 2.3 \times 10^{-13}$

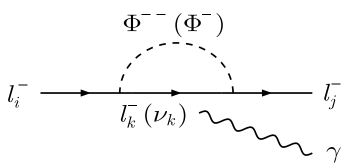

Fig. 1. Feynman diagrams contributing to the radiative decay $l_i^- \to l_j^- \gamma$ due to the charged scalars Φ^{--} (Φ^-)

Using (4), the expression of the branching ratio $\mathrm{Br}(l_i^- \to l_j^- \gamma)$ can be written at leading order:

$$\mathrm{Br}(l_i^- \to l_j^- \gamma) = \frac{\alpha_e}{96\pi G_\mathrm{F}^2} \sum_{k=\tau,\mu,e} (Y_{ik} Y_{kj}^*)^2 \times \left[\frac{3\delta_{ki(j)} + 1}{M_{\Phi^{--}}^2} + \frac{1}{M_{\Phi^-}^2} \right]^2 \mathrm{Br}(l_i \to e\nu_e \overline{\nu}_i) \,. \tag{5}$$

Here α_e is the fine structure constant, and G_F is the Fermi constant. The factor $3\delta_{ki(j)}$ means that, when the internal lepton is the same as one of the leptons l_i and l_j, the contributions of Φ^{--} to $\mathrm{Br}(l_i^- \to l_j^- \gamma)$ are four times those for $k \neq i$ and j. $M_{\Phi^{--}}$ and M_{Φ^-} are the masses of the scalars Φ^{--} and Φ^-, respectively. In the LH model, the scalar mass is generated through the Coleman–Weinberg mechanism, and the scalars Φ^{--}, Φ^- and Φ^0 are degenerate at the lowest order [9]. Thus, we can assume that $M_{\Phi^{--}} = M_{\Phi^-}$ and write the branching ratio as

$$\mathrm{Br}(l_i^- \to l_j^- \gamma) = \frac{\alpha_e}{96\pi G_\mathrm{F}^2 M_\Phi^4} \sum_{k=\tau,\mu,e} (Y_{ik} Y_{kj}^*)^2 [3\delta_{ki(j)} + 2]^2 \mathrm{Br}(l_i \to e\nu_e \overline{\nu}_i) \,. \tag{6}$$

In particular, for the decay process $\mu^- \to e^- \gamma$, we obtain the following expression for the branching ratio $\mathrm{Br}(\mu^- \to e^- \gamma)$:

$$\mathrm{Br}(\mu^- \to e^- \gamma) = \frac{\alpha_e}{96\pi G_\mathrm{F}^2 M_\Phi^4} \left[25(Y_{\mu e} Y_{ee}^*)^2 + 25(Y_{\mu\mu} Y_{\mu e}^*)^2 + 4(Y_{\mu\tau} Y_{\tau e}^*)^2 \right] . \tag{7}$$

From the above equations, we can see that the LFV process $l_i \to l_j \gamma$ cannot be able to constrain Y_{ij} independently. However, if we assume $Y_{ik} = Y$ for $i = k$ (Y is the flavor-diagonal (FD) coupling constant) and $Y_{ik} = Y'$ for $i \neq k$ (Y' is the flavor-mixing (FX) coupling constant), then we can obtain constraints on the combination of the free parameters Y, Y' and M_Φ. In terms of observability, the most stringent constraint should come from the current experimental upper limits on the branching ratio $\mathrm{Br}(\mu \to e\gamma)$. Thus, in Fig. 2, we have shown the FD coupling constant Y as a function of the mass parameter M_Φ for $Y' = 1 \times 10^{-2}$, 1×10^{-3} and 1×10^{-4}. From Fig. 2, one can see the upper limit on Y to strongly depend on the values of M_Φ and Y'. For $M_\Phi \leq 2000$ GeV and $Y' \geq 1 \times 10^{-4}$, we must have $Y \leq 64$.

In the LH model, the LFV processes $l_i \to l_j l_k l_k$ can be generated at tree level through the exchange of the doubly charged scalar $\Phi^{\pm\pm}$, as depicted in Fig. 3.

The expressions of the branching ratios for the processes $l_i^- \to l_j^+ l_k^- l_k^-$ are given by [23–25]:

$$\mathrm{Br}(\mu^- \to e^+ e^- e^-) = \frac{\mid Y_{\mu e} Y_{ee}^* \mid^2}{16 G_\mathrm{F}^2 M_\Phi^4} \,, \tag{8}$$

$$\mathrm{Br}(\tau^- \to e^+ e^- e^-) = \frac{\mid Y_{\tau e} Y_{ee}^* \mid^2}{16 G_\mathrm{F}^2 M_\Phi^4} \mathrm{Br}(\tau \to e\nu_e \overline{\nu}_\tau) \,, \tag{9}$$

$$\mathrm{Br}(\tau^- \to \mu^+ e^- e^-) = \frac{\mid Y_{\tau\mu} Y_{ee}^* \mid^2}{32 G_\mathrm{F}^2 M_\Phi^4} \mathrm{Br}(\tau \to e\nu_e \overline{\nu}_\tau) \,, \tag{10}$$

$$\mathrm{Br}(\tau^- \to e^+ \mu^- \mu^-) = \frac{\mid Y_{\tau e} Y_{\mu\mu}^* \mid^2}{32 G_\mathrm{F}^2 M_\Phi^4} \mathrm{Br}(\tau \to e\nu_e \overline{\nu}_\tau) \,, \tag{11}$$

$$\mathrm{Br}(\tau^- \to \mu^+ \mu^- \mu^-) = \frac{\mid Y_{\tau\mu} Y_{\mu\mu}^* \mid^2}{16 G_\mathrm{F}^2 M_\Phi^4} \mathrm{Br}(\tau \to e\nu_e \overline{\nu}_\tau) \,. \tag{12}$$

Certainly, up to one loop, the LFV processes $l_i \to l_j l_k l_k$ get additional contributions from the processes $l_i \to l_j \gamma^* \to$

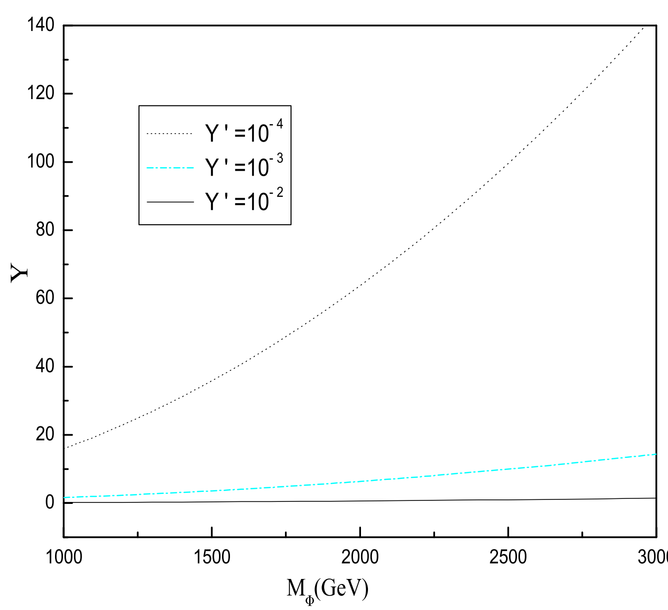

Fig. 2. The FD coupling constant Y as a function of the scalar mass M_Φ for different values of the FX coupling constant Y'

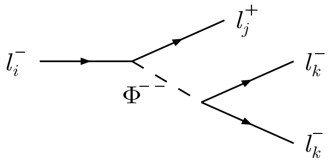

Fig. 3. Tree level Feynman diagram for the LFV processes $l_i^- \to l_j^+ l_k^- l_k^-$ mediated by the doubly charged scalar Φ^{--}

$l_j l_k l_k$. Thus, the charged scalars $\Phi^{\pm\pm}$ and $\Phi^{\pm}$ have contributions to the LFV processes $l_i \to l_j l_k l_k$ at one loop. However, compared with the tree level contributions, they are very small and can safely be neglected.

The LFV processes $l_i \to l_j l_k l_k$ also cannot give constraints on the coupling constants Y_{ij} independently, but they may be able to constrain the combination $| Y_{ij} Y_{kk}^\dagger |^2 / M_\Phi^4$. Our numerical results are given in Table 1.

In the following section, we will take into account these constraints coming from the LFV processes $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_k$; we estimate the contributions of the doubly charged scalars $\Phi^{\pm\pm}$ to the processes $e^\pm e^\pm \to l_i^\pm l_j^\pm$ and $e^+e^- \to l_i^\pm l_j^\pm$, and we discuss the possibility of detecting the signals for the doubly charged scalars $\Phi^{\pm\pm}$ at the ILC experiments.

4 The doubly charged scalars $\Phi^{\pm\pm}$ and the LFV processes $e^\pm e^\pm \to l_i^\pm l_j^\pm$ and $e^+e^- \to l_i^\pm l_j^\pm$

In general, the doubly charged scalars cannot couple to quarks, and their couplings to leptons break the lepton number by two units, leading to a distinct signature, namely a pair of same-sign leptons. The discovery of a doubly charged scalar would have important implications for our understanding of the Higgs sector and, more importantly, of what lies beyond the SM. This fact has triggered more elaborate theoretical calculations in the framework of some specific models beyond the SM to see whether signatures of this kind of new particles can be detected in future high energy experiments. For example, the production and decay of the doubly charged scalars and their possible signals at the ILC have been extensively studied in [26–32]. In this section, we will consider the contributions of the doubly charged scalars $\Phi^{\pm\pm}$ predicted by the LH model to the processes $e^\pm e^\pm \to l_i^\pm l_j^\pm$ and $e^+e^- \to l_i^\pm l_j^\pm$ (l_i or $l_j \neq e$). The processes $e^\pm e^\pm \to l_i^\pm l_j^\pm$ can be seen as subprocesses of the processes $e^+e^- \to l_i^\pm l_j^\pm$. For example, the doubly charged scalar Φ^{--} generates contributions to the process $e^+e^- \to l_i^- l_j^-$ through the subprocess $e^-e^- \to l_i^- l_j^-$, as shown in Fig. 4.

Using (4), the expression for the cross section for the subprocess $e^-e^- \to l_i^- l_j^-$ can easily be written

$$\widehat{\sigma}(\widehat{s}) = \frac{Y_{ee}^2 Y_{ij}^2}{8\pi} \frac{\widehat{s}}{(\widehat{s} - M_\Phi^2)^2 + M_\Phi^2 \Gamma_\Phi^2} \,. \tag{13}$$

Here $\sqrt{\widehat{s}}$ is the center-of-mass (C.M.) energy of the subprocess $e^-e^- \to l_i^- l_j^-$. Γ_Φ is the total decay width of the

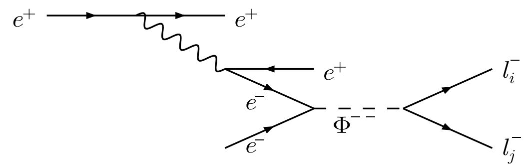

Fig. 4. Main Feynman diagram for the processes $e^+e^- \to l_i^- l_j^-$ predicted by Φ^{--}

doubly charged scalar Φ^{--}, which has been given by [9] in the case of the triplet scalars ($\Phi^{\pm\pm}, \Phi^{\pm}$, and Φ^0) becoming degenerate at lowest order with a common mass M_Φ:

$$\begin{aligned} \Gamma_\Phi &= \sum_{ij} \Gamma(\Phi^{--} \to l_i^- l_j^-) + \Gamma(\Phi^{--} \to W_L^- W_L^-) \\ &\quad + \Gamma(\Phi^{--} \to W_T^- W_T^-) \\ &\approx \frac{M_\Phi}{8\pi}[3Y^2 + 6Y'^2] + \frac{\nu'^2 M_\Phi^3}{2\pi\nu^4} + \frac{g^4 \nu'^2}{4\pi M_\Phi} \,, \end{aligned} \tag{14}$$

where $Y = Y_{ij}$ $(i = j)$ is the FD coupling constant, $Y' = Y_{ij}$ $(i \neq j)$ is the FX coupling constant. In the above equation, the final-state masses have been neglected compared to the mass parameter M_Φ. It has been shown that, for $\nu' < 1 \times 10^{-5}$, the main decay modes of Φ^{--} are $l_i^- l_j^-$. Furthermore, the FX coupling constant Y' is subject to very stringent bounds from the LFV process $\mu \to eee$. In this case, the decay width Γ_Φ can approximately be written as

$$\Gamma_\Phi \approx \frac{3 M_\Phi Y^2}{8\pi} \,. \tag{15}$$

Considering the current bounds on the neutrino mass [12–14], we should have

$$Y_{ij} \nu' \sim 10^{-10}\,\text{GeV} \,, \tag{16}$$

so $\nu' < 1 \times 10^{-5}$ leads to $Y_{ij} > 1 \times 10^{-5}$, which does not conflict with the most stringent constraint from the LFV process $\mu \to eee$. Thus, in our numerical calculation, we will take (15) as the total decay width of Φ^{--}.

Using the method of the equivalent particle approximation [33–35], the effective cross section for the process $e^+e^- \to l_i^- l_j^-$ can be approximately written as [30–32]

$$\sigma(E_{e^+}, s) = \int_{x_{\min}}^1 \mathrm{d}x F_{e^+}^{e^-}(x, E_{e^+}) \widehat{\sigma}(\widehat{s}) \,, \tag{17}$$

where $\widehat{s} = xs$ and $x_{\min} = (m_{l_i} + m_{l_j})^2/s$. $F_{e^+}^{e^-}(x, E_{e^+})$ is the equivalent electron distribution function of the initial positron, which gives the probability that an electron with energy $E_{e^-} = xE_{e^+}$ is emitted from a positron beam with energy E_{e^+}. The relevant expression can be written as [36]

$$\begin{aligned} F_{e^+}^{e^-}(x, E_{e^+}) &= \frac{\alpha_e^2}{8\pi^2 x} \left[\ln\left(\frac{E_{e^+}}{m_e}\right)^2 - 1 \right]^2 \\ &\quad \times \left[\frac{4}{3} + x - x^2 - \frac{4}{3}x^3 + 2x(1+x)\ln x \right] . \end{aligned} \tag{18}$$

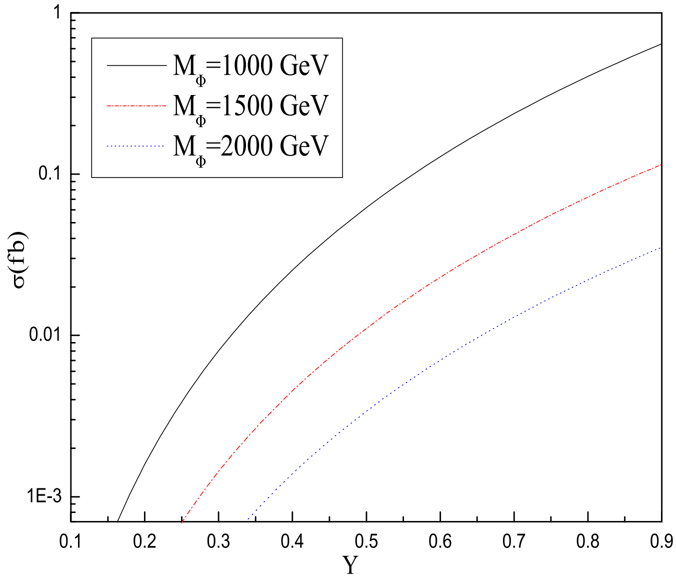

Fig. 5. The cross section $\widehat{\sigma}(\widehat{s})$ as a function of Y for three values of the mass M_Φ

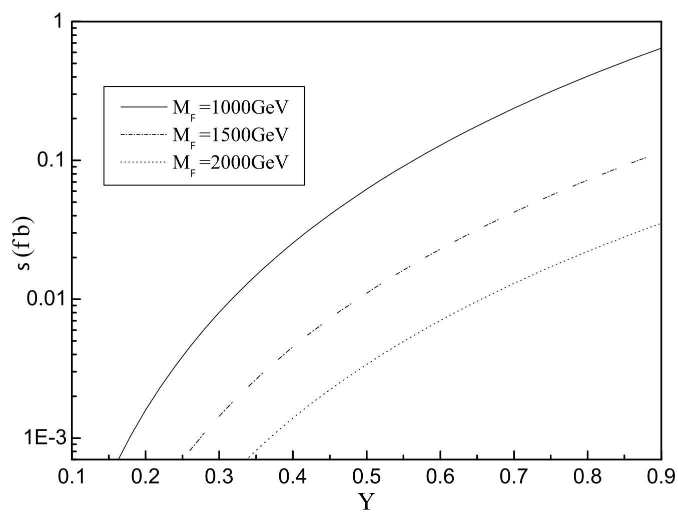

Fig. 6. Same as Fig. 5, but for $\sigma(s)$

In Figs. 5 and 6, we plot the production cross sections $\widehat{\sigma}(\widehat{s})$ and $\sigma(s)$ for the processes $e^-e^- \to \mu^-\mu^-$ and $e^+e^- \to \mu^-\mu^-$ as a function of the FD coupling constant Y, respectively. In these figures, we have assumed $0.15 \leq Y \leq 0.9$ and we have taken $\sqrt{s} = 500$ GeV and $M_\Phi = 1.0$ TeV, 1.5 TeV and 2.0 TeV. From Figs. 5 and 6 one can see that the values of $\widehat{\sigma}(\widehat{s})$ and $\sigma(s)$ are strongly dependent on the value of the FD coupling constant $Y(Y_{ee})$. For $Y \geq 0.7$ and $M_\Phi \leq 1.5$ TeV, the values of the subprocess cross section $\widehat{\sigma}(\widehat{s})$ and the effective cross section $\sigma(s)$ are larger than 1.1×10^2 fb and 4.3×10^{-2} fb, respectively.

The signal of the doubly charged scalar Φ^{--} given by the process $e^+e^- \to \mu^-\mu^-$ is so distinctive and is so SM background free that discovery would be signaled by even a few events. In Fig. 7, we plot the discovery region in the Y–M_Φ plane at 95% confidence level (C.L.) for seeing $5\mu^-\mu^-$

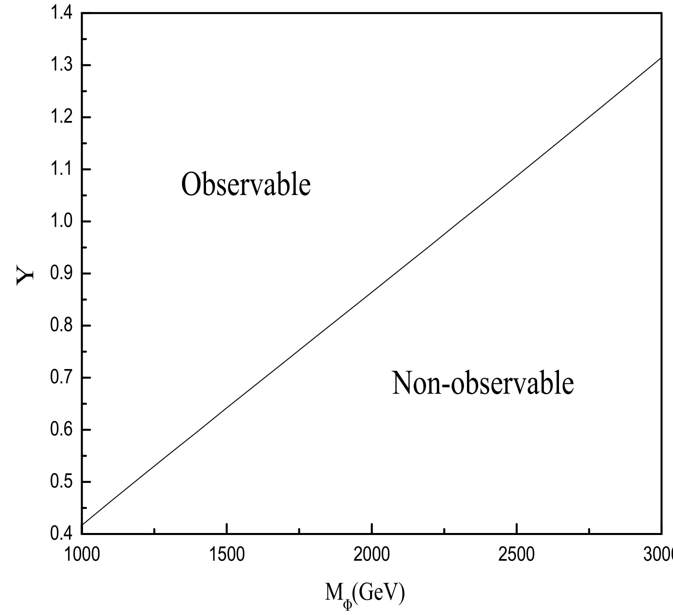

Fig. 7. Discovery region in the Y–M_Φ plane at 95% C.L. for seeing $5\mu^-\mu^-$ events

events, in which we have assumed the future ILC to run with C.M. energy $\sqrt{s} = 500$ GeV and a yearly integrated luminosity of $\mathcal{L} = 500\ \mathrm{fb}^{-1}$ [37–40]. From this figure, one can see that, in a wide range of the parameter space, the signals of Φ^{--} should be detected in future ILC experiments.

The doubly charged scalar Φ^{--} can also have contributions to the LFV processes $e^+e^- \to \tau^-\mu^-, \tau^-e^-$, and μ^-e^-. However, the experimental upper limits on the LFV processes $\tau \to \mu ee$, $\tau \to eee$ and $\mu \to eee$ can give severe constraints on the combination $\mid Y_{ij}Y^\dagger_{kk} \mid^2 / M^4_\Phi$, which makes the production cross sections of these processes very small. For example, even if we take $Y = 1$ and $M_\Phi \leq 2$ TeV, the production cross sections $\sigma(\tau\mu)$, $\sigma(\tau e)$ and $\sigma(\mu e)$ are smaller than 6.9×10^{-3} fb, 2.1×10^{-3} fb and 1.9×10^{-9} fb, respectively. Thus, it is very difficult to detect the signals of Φ^{--} via the processes $e^+e^- \to l_i^- l_j^-$ $(i \neq j)$ in future ILC experiments.

Certainly, the doubly charged scalar Φ^{++} gives contributions to the processes $e^+e^+ \to l_i^+ l_j^+$ and $e^+e^- \to l_i^+ l_j^+$. Similar to the above calculation, we can give the values of the production cross sections for these processes. We find that the cross section $\sigma(l_i^+ l_j^+)$ is equal to the cross section $\sigma(l_i^- l_j^-)$. Thus, the conclusions for the doubly charged scalar Φ^{--} also apply to the doubly charged scalar Φ^{++}.

5 Conclusions

To solve the so-called hierarchy or fine tuning problem of the SM, the little Higgs theory was proposed as a kind of model for EWSB, accomplished by a naturally light Higgs boson. The LH model is one of the simplest and one of the phenomenologically viable models. In the LH model, neutrino masses and mixings may be generated by coupling the scalar triplet Φ to the leptons in a $\Delta L = 2$ interaction whose magnitude is proportional to the triplet VEV ν' multiplied by the Yukawa coupling constant Y_{ij}, without invoking a right handed neutrino. This scenario predicts the existence of the doubly charged scalars $\Phi^{\pm\pm}$. For smaller values of ν' i.e. $\nu' \leq 1 \times 10^{-5}$, the doubly charged scalars $\Phi^{\pm\pm}$ have a large flavor changing coupling to leptons, which can generate significantly contributions to some LFV processes and can give characteristic signatures in future high energy experiments.

In this paper, we first consider the LFV processes $l_i \to l_j\gamma$ and $l_i \to l_j l_k l_k$ in the context of the LH model. For the LFV process $l_i \to l_j\gamma$, it involves all of the FX coupling constants Y_{ij} $(i \neq j)$, and we cannot give simple constraints on the free parameters Y_{ij} and M_Φ. Thus, for the fixed values of the FX coupling constant $Y' = Y_{ij}$ $(i \neq j)$, we take into account the current experimental upper limit of the LFV $\mu \to e\gamma$ and plot the FD coupling constant $Y = Y_{ij}$ $(i = j)$ as a function of the mass parameter M_Φ. Our numerical results show that the upper limit on Y is strongly dependent on the free parameters M_Φ and Y'.

Using the present experimental upper limits on the branching ratios $\text{Br}(l_i \to l_j l_k l_k)$, we obtain the constraints on the combination $\mid Y_{ij} Y_{kk}^* \mid^2 / M_\Phi^4$. We find that the most stringent constraint comes from the LFV process $\mu \to eee$. In all of the parameter space, we must have $\mid Y_{\mu e} Y_{ee}^* \mid^2 / M_\Phi^4 \leq 2.2 \times 10^{-19}\ \text{GeV}^{-4}$.

The characteristic signals of the processes $e^+e^- \to l_i^\pm l_j^\pm$ are same-sign dileptons or two same-sign different flavor leptons, which are SM background free and offer excellent potential for the discovery of the doubly charged scalar. To see whether the doubly charged scalar Φ^{--} can be detected in future ILC experiments, we discuss the contributions of Φ^{--} to the processes $e^-e^- \to l_i^- l_j^-$ and $e^+e^- \to l_i^- l_j^-$. We find that the triplet scalar Φ^{--} may give significantly contributions to the processes $e^+e^- \to l_i^- l_i^-$. In a wide range of the parameter space of the LH model, the possible signals of Φ^{--} might be observed in future ILC experiments. However, the production cross sections of the LFV processes $e^+e^- \to l_i^- l_j^-$ $(i \neq j)$ mediated by Φ^{--} are very small. The contributions of the triplet scalar Φ^{++} to the processes $e^+e^- \to l_i^+ l_j^+$ are equal to those of Φ^{--} for the processes $e^+e^- \to l_i^- l_j^-$, Thus, our conclusions also apply to the doubly charged scalar Φ^{++}.

Some popular models beyond the SM predict the existence of doubly charged scalars, which generally have lepton number and lepton flavor changing couplings to leptons and might produce distinct experimental signatures in current or future high energy experiments. Their observation would signal physics outside the current paradigm and further test the new physics models. The search for this kind of new particles has been one of the important goals of the high energy experiments [41–45]. Thus, the possibly signals of the doubly charged scalars $\Phi^{\pm\pm}$ predicted by little Higgs models should be further studied in the future.

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